Soft Robotic Simulation using Physics-Informed Koopman Operators in the Low-data Limit

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Introduction

Soft robotics are capable of producing biocompatible motions that are conformable to humanlike functions, and as such, are more capable of safely working alongside humans. Existing methods require large amounts of data to be able to have high enough accuracy for control. To address this, **we propose a physics-informed data-driven method that has higher accuracy in the low-data limit than existing methods.**

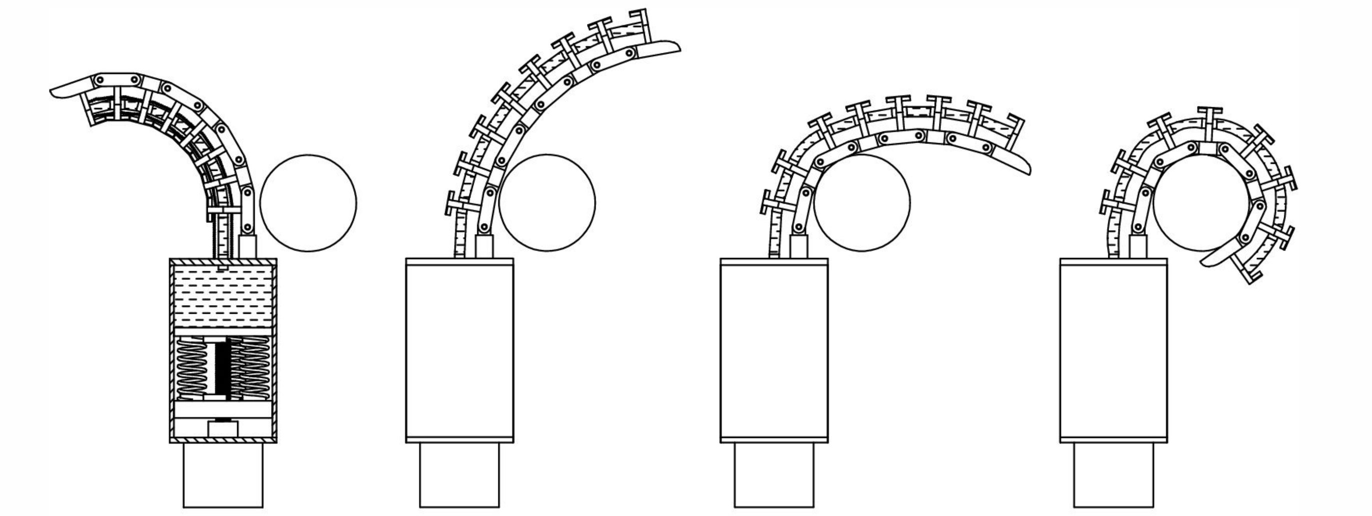

Figure 1: A simple schematic of a continuum robot undergoing actuation. Adapted from [Wei, 2018]

Mathematical Formulation

Inspired by the work done by [Proctor, 2018] and contributors, we adopt a Koopman theoretic approach to data-driven dynamics. Given a dynamical system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{h}(\mathbf{x}, \mathbf{u})$$

1. Solve for the continuous-time passive system
$$\mathcal{L}_1 := J(X)\Theta^{\dagger}(X)$$
2. Solve the discrete-time perturbations due to control inputs
$$\mathcal{L}_2 := \left(\Theta(X') - e^{\Delta t \mathcal{L}_1}\right)\Theta^{\dagger}(X, U)$$
3. Propagate forward in time
$$\Theta(\mathbf{x}_{k+1}) \approx e^{\Delta t \mathcal{L}_1}\Theta(\mathbf{x}_k) + e^{\Delta t \mathcal{L}_1}\int_0^{\Delta} t e^{-\tau \mathcal{L}_1}\mathcal{L}_2 \Theta(\mathbf{x}_k, \mathbf{u}_k)\mathrm{d}\tau$$

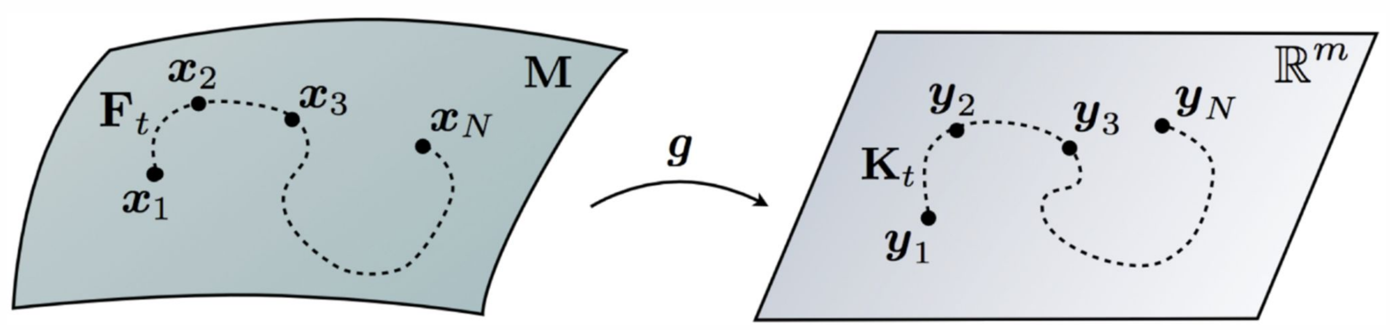

Figure 2: Dynamics in the lifted space. From [Proctor, 2018]

Numerical Results

Slow Manifold Example [Proctor, 2018]

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \mu x_1 \\ \lambda(x_2 - x_1^2) \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} u$$

Method	Dictionary for state	Dictionary for state and control	# of Functions
Bilinear EDMDc	N/A	Order 2 monomials	5
Linear EDMDc	N/A	Order 2 monomials	5
PI EDMDc	Order 2 monomials	Identity function over control	6

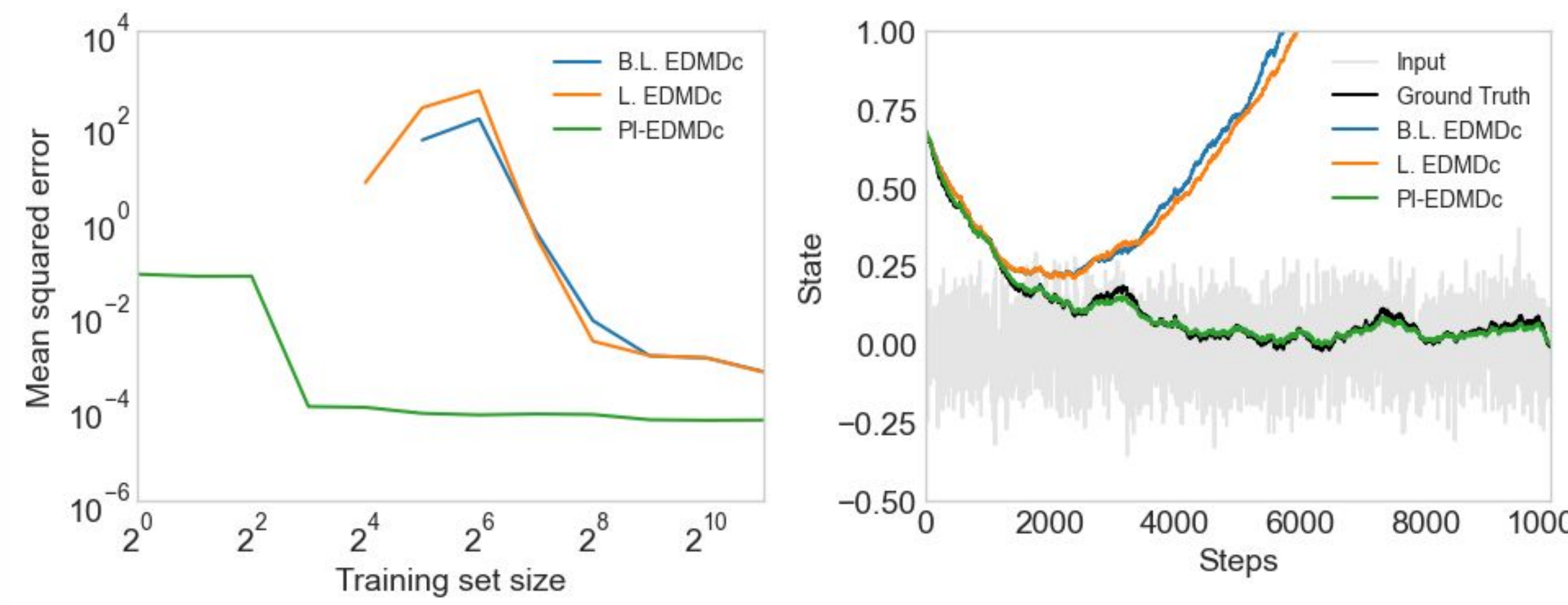

Figure 3: a) Mean squared error as a function of training set size. b) A sample trajectory of 10000 steps with predicted trajectories trained on 2048 data points.

Heat Equation along a Rod

$$\frac{\partial}{\partial t}u(x,t) = k\frac{\partial^2}{\partial t^2}u(x,t) + \delta\sin(\omega x)$$

Method	Dictionary for state	Dictionary for state and control	# of Functions
Bilinear EDMDc	N/A	Order 3 polynomials	193
Linear EDMDc	N/A	Order 3 polynomials	193
PI-EDMDc	Order 3 polynomials	Identity function over control	44

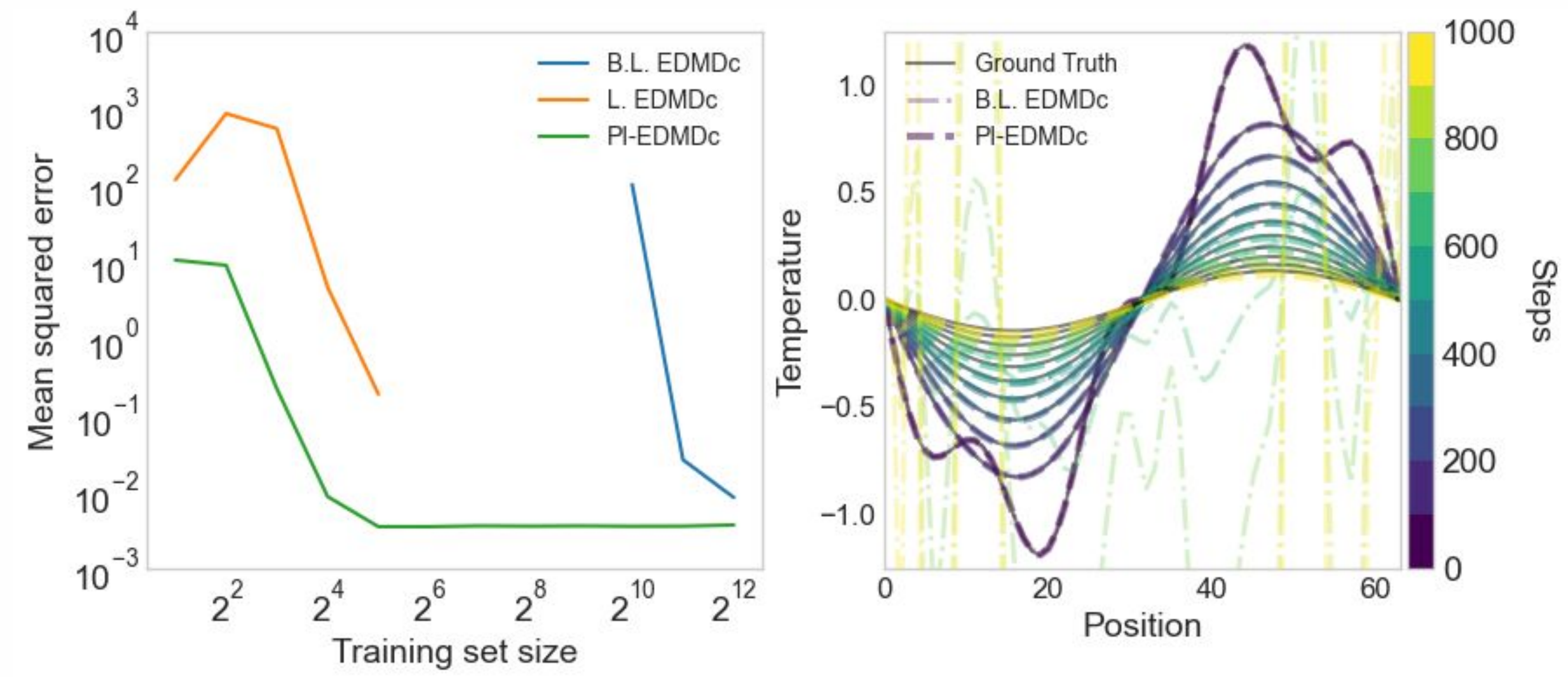

Figure 4: a) Mean squared error as a function of training set size. b) Performance of PI-EDMDc compared to bilinear EDMDc over 1000 steps trained on 1024 data points.

Tendon Robot with Flexible Backbone

Simulations are conducted as in [Till, 2019]

$$\boldsymbol{p}_s = \boldsymbol{R}\boldsymbol{v},\ \boldsymbol{p}_t = \boldsymbol{R}\boldsymbol{q} \qquad \boldsymbol{m}_s = \partial_t(\boldsymbol{R}\rho\boldsymbol{J}\boldsymbol{\omega}) - \widehat{\boldsymbol{p}}_s\boldsymbol{n} - \boldsymbol{l}$$
$$\boldsymbol{R}_s = \boldsymbol{R}\widehat{\boldsymbol{u}},\ \boldsymbol{R}_t = \boldsymbol{R}\widehat{\boldsymbol{\omega}} \qquad \boldsymbol{q}_s = \boldsymbol{v}_t - \widehat{\boldsymbol{u}}\boldsymbol{q} + \widehat{\boldsymbol{\omega}}\boldsymbol{v}$$
$$\boldsymbol{n}_s = \rho A\boldsymbol{R}\left(\widehat{\boldsymbol{\omega}}\boldsymbol{q} + \boldsymbol{q}_t\right) - \boldsymbol{f} \qquad \boldsymbol{\omega}_s = \boldsymbol{u}_t - \widehat{\boldsymbol{u}}\boldsymbol{\omega}$$

Method	Dictionary for state	Dictionary for state and control	# of Functions
Bilinear EDMDc	N/A	Random Fourier (RF), γ=1	204
Linear EDMDc	N/A	RF, γ=1	204
PI-EDMDc	RF, γ=1	Order 1 monomials	332

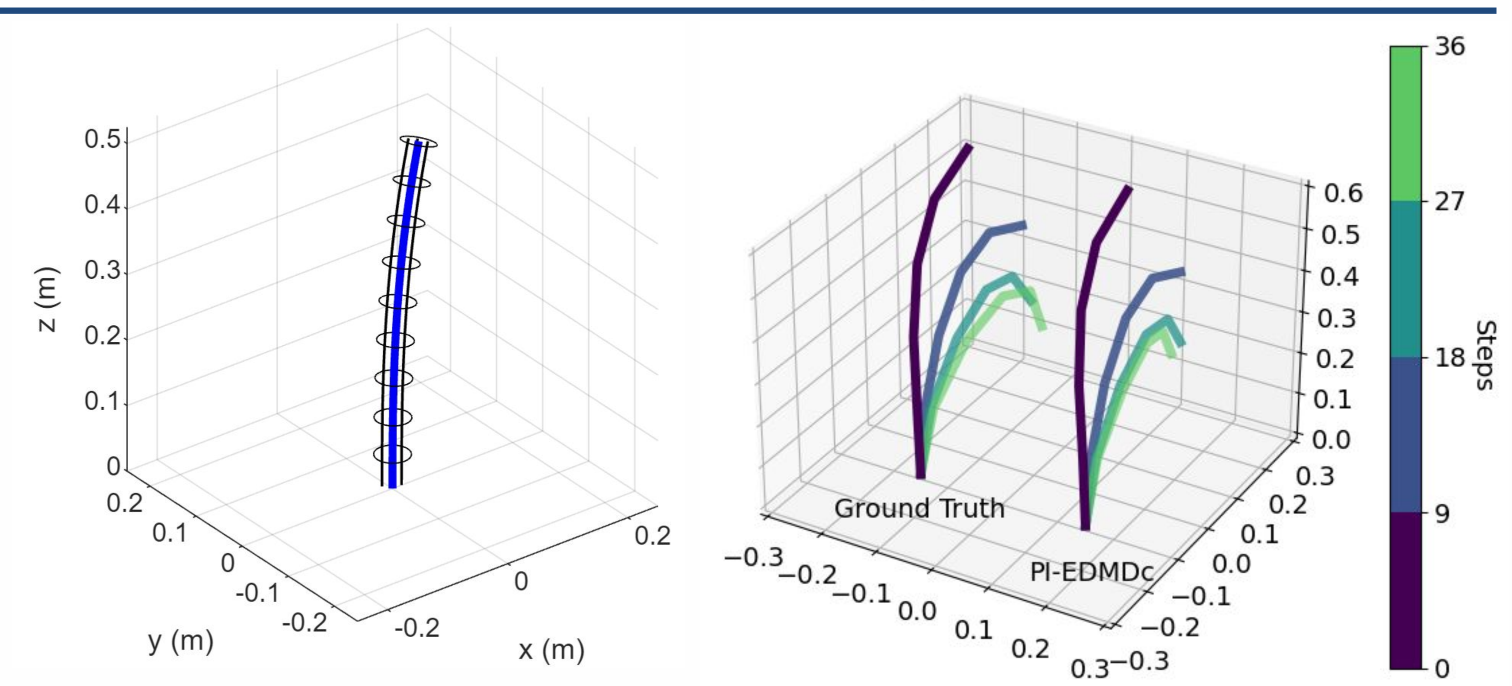

Figure 5: a) Simulated rendering of tendon robot. b) Performance of PI-EDMDc compared to the ground truth trained on 15,000 data points.

Discussion and Conclusions

Overall, our method, **P**hysics-**I**nformed **E**xtended **D**ynamic **M**ode **D**ecomposition with **C**ontrol (PI-EDMDc), is able to effectively estimate forward in time even in the low-data limit.

We have shown:

- Improved accuracy in the low-data limit (2 orders of magnitude)
- Improved generalization performance in the low-data limit
- Effective simulation of realistic cable driven soft robots (such as in Figure 6)
- High applicability of the method to a variety of systems

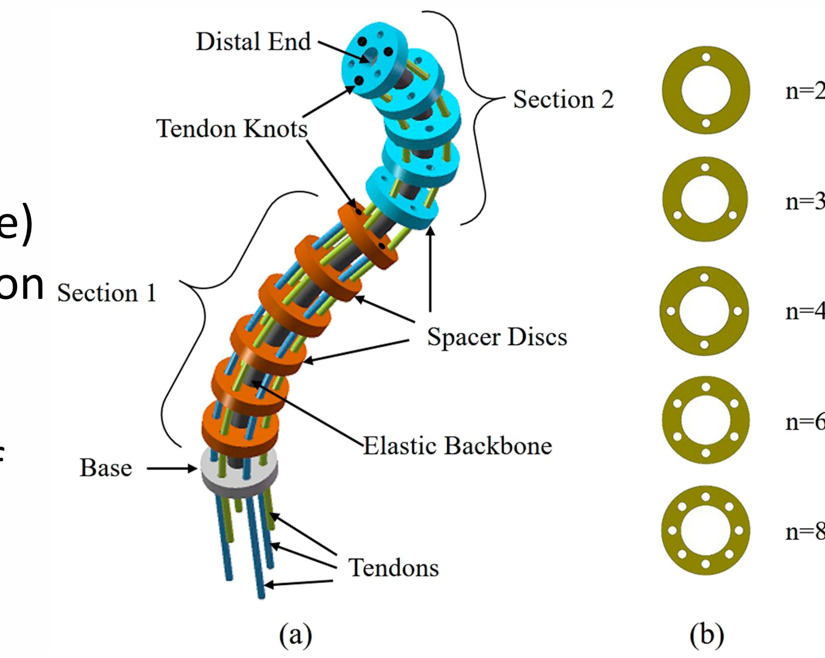

Figure 6: Multi-section tendon robot used in surgical settings. Adapted from [Li, 2017]

Future Work

Forward simulation is the first step towards being able to control a continuum robot. As Koopman theoretic approaches have ideal properties in terms of identifying optimal control inputs, we hope to, in the near future:

- Develop a control algorithm using the developed method
- Construct a physical prototype for validating this method

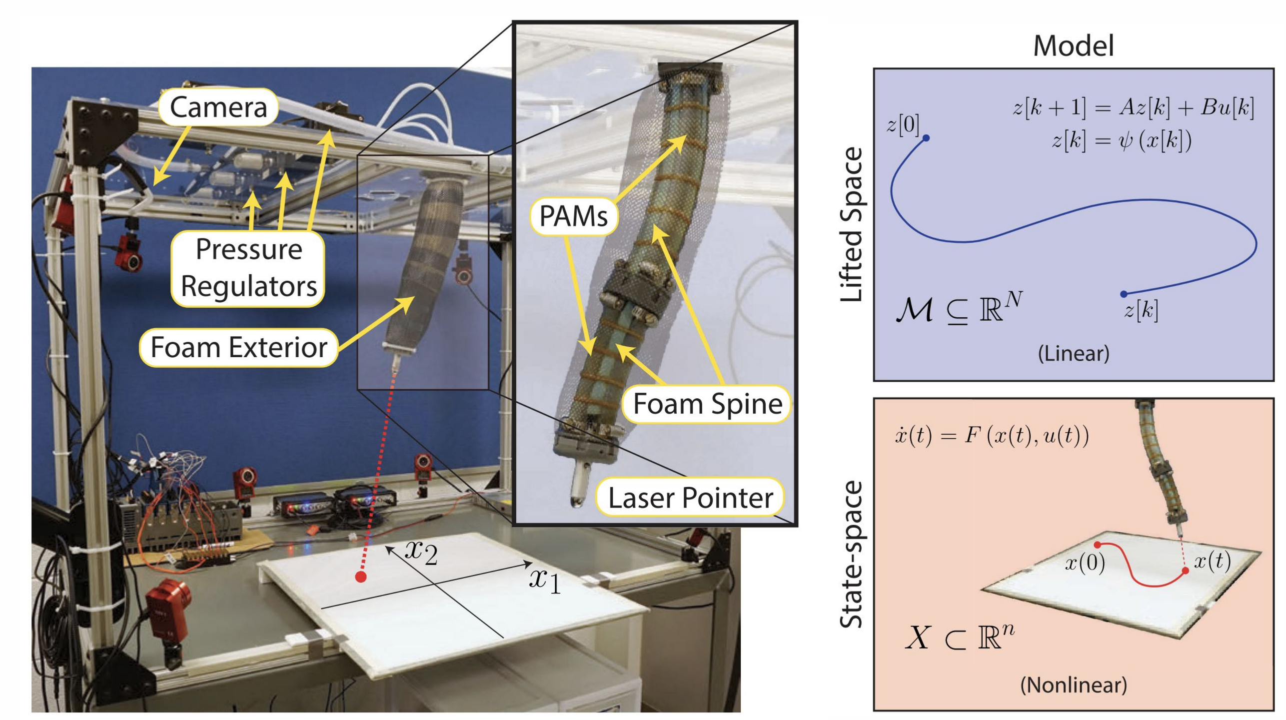

Figure 7: Experimental setup for a soft robot. Adapted from [Bruder, 2019]

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